Due to the increasing number of natural and technogenic disasters the development of geological environment monitoring system is actual using modern mathematical tools and information technology. The local monitoring of potentially dangerous objects is an important part of the overall environment monitoring system. Complex geophysical research was conducted on Rivne NPP area. Among these monitoring observations radioisotope study of soil density and humidity near the perimeter of buildings is of the greatest interest.

In this case a problem was occurred to supplement simulated data that were received at the control of chalky strata density changes at the research industrial area with use of radioisotope methods on a grid that included 29 wells. This problem was solved in this work by statistical simulation method that provides the ability to display values (random field on a plane) in any point of the monitoring area. The chalk strata average density at the industrial area was simulated using the built model and the involvement of the Cauchy type correlation function.

This paper analyses the method and the model and procedure were developed with enough adequate data for Cauchy function. The method and algorithm were developed and examples of karst-suffusion phenomena statistical simulation were given in the problem of density chalk strata monitoring at the Rivne NPP area. The statistical model of average density chalk strata distribution was built in the plane and statistical simulation algorithm was developed using Cauchy function on the basis of spectral decomposition. The research subject realizations were obtained with required detail and regularity at the observation grid based on the developed software. Statistical analysis of the numerical simulation results was done and tested its adequacy.

Keywords: Statistical simulation, function Cauchy, spectral decomposition, conditional maps.
strata density changes at the research industrial area with use of radioisotope methods on a grid that included 29 wells. Schematic representation of the measurement results at the object that was investigated, and the well locations are shown on Fig. 1. These data are obviously not enough to represent the overall picture of the chalk strata, where due to the aggressive water action the karst-suffusion processes were significantly intensified.

Fig.1. Observation points and chalk strata averaged density at industrial area of Rivne NPP

This problem was solved in work [5] by statistical simulation method that provides the ability to display values (random field on a plane) in any point of the monitoring area. The chalk strata average density at the industrial area was simulated using the built model and the involvement of the Bessel type correlation function.

This work continues development of methods for statistical simulation, involving Cauchy correlation function that is well-known in Geostatistics [1,3].

Before using developed mathematical tools to radioisotope research real data of soil density at the observation area their preliminary preparation and statistical analysis was carried out.

At the data preparation stage the measurement errors were excluded (for the operator, the equipment, observation conditions, etc.). With this purpose the following things were made:

- Firstly, the top layer was rejected (about 10 meters), as it consists of mixed bulk soil;
- Secondly, well data were aggregated for the different years to the average, which was calculated for three years in anchor wells (the reference loam layer was selected that lies above the chalk layer). For loam layer the average value for three years was calculated and then the corresponding corrections were put into data with a "+" or "-" sign. Such processing was done by STATISTICA software.

This operation was done for data array of density chalk strata in 1984-2002 years’ for 29 wells at Rivne NPP industrial area and depth is 28 m below the surface.

The method of solving the problem. Data of density chalky strata was divided into deterministic and random components. Deterministic function can be selected by the method of approaching the minimum curve (separation of the trend). The difference between the map of input density values and the trend is in most cases a homogeneous isotropic random field. With the assumption that the input data is a random field \( \eta(\mathbf{x}) \), then we express them through a random component \( \xi(\mathbf{x}) \) (so-called "noiise" random field) and trend \( f(\mathbf{x}) \) as a deterministic function as follows:

\[
\eta(\mathbf{x}) = f(\mathbf{x}) + \xi(\mathbf{x}).
\]

Thus, the problem was reduced to simulation of random component \( \xi(\mathbf{x}) \), which in most cases is a homogeneous and isotropic.

Consider the same approach as in [2, 6, 7]. We use the method of statistical simulation of random fields, which are homogenous and isotropic, based on their spectral decomposition. By means of the obtained values of realizations, this technique allows to find the perfect image of these isotropic fields in the whole observation interval.

It is necessary to make the statistical analysis to build the model and procedure of statistical data simulation at observation area. If the verified data have distribution density with approximately Gaussian type, then procedure can be used, which is developed in [2, 6, 7], to generate on the computer realizations of the simulated data by means of standard normal random variable sequences.

At first the distribution is determined. The preliminary statistical analysis of data shows that the distribution histogram of chalky strata density at the Rivne industrial area (29 boreholes) approximately has Gaussian distribution (Fig. 2).

The use of authors’ techniques of statistical simulation implies preliminary statistical data processing to determine its statistical characteristics: the mathematical expectation and the correlation function. If the hypothesis of Gaussian distribution of the investigated field is confirmed, then the mathematical expectation and the correlation function completely define this field and give us the opportunity to build the adequate statistical model, which is based on spectral decomposition of random functions. The principles of constructing the models and procedures are described below.
Fig. 2. Histogram of the chalky strata density (averaged data for all years of observation):
1 – the number of observations in a separate range of density; 2 – theoretical Gaussian curve

Then the statistical model was chosen for the data correlation function for distribution of chalky strata density in the flat observation area. This function is defined by comparing the mean square approximation of the empirical and theoretical variograms. As result the input data was most adequately described by means of two types of correlation functions: the Bessel function (1) at the value of parameter \(a=5\) and the Cauchy function (2) at the value of parameter \(a=1\):

\[
B(\rho) = J_0(\rho a), \quad a = 5; \tag{1}
\]

\[
B(\rho) = \frac{a^4}{(a^2 + \rho^2)^\frac{a}{2}}, \quad a = 1. \tag{2}
\]

The variogram of input data of chalky strata density, corresponding to the Cauchy correlation function (2), was built by using the R software and geoR package. The plot was presented at Figure 3, according to the Cauchy correlation function (2) variogram of the random component of investigation data.

Note that the generalized Cauchy model is:

\[
B(\rho) = \left(1 + \frac{\rho^2}{a^2}\right)^{-\nu}, \quad \nu > 0, \quad a > 0. \tag{3}
\]

The generalized Cauchy model (3) was studied by A. M. Yaglom [12], T. Gneiting [3] who considered the Cauchy function at the values of the parameter \(\nu = 1/2, 3/2, 5/2, 7/2\).
Fig. 4. The Cauchy function at parameter values $a = 1$ and $\nu = 0.25, 0.5, 1, 2, 5$ and $10$

Let us find the spectral density, which corresponds to the Cauchy correlation function (2), by using the formula 6.565 (4) and 8.486 (16) [5]. Formula 6.565 (4) is mentioned below as:

$$\int_0^\infty \frac{J_0(bx)}{(a^2+x^2)^{b+1}}dx = \frac{a^{-b}b^{b-1}}{2^b\Gamma(b+1)}K_{\nu}(ab), a > 0, b > 0, \Re \nu < \Re (2\mu + 3/2).$$

and the ratio 8.486 (16) is written as follows:

$$K_\nu(\nu) = K_{\nu}(\nu).$$

Then such spectral density is calculated by the following formula:

$$f(\lambda) = \lambda \int_0^\infty xJ_\nu(\lambda x)B(x)dx = \lambda \int_0^\infty xJ_\nu(\lambda x)K_\nu(\nu)dx,$$

Thus the spectral density, which is corresponding to the Cauchy correlation function (2), is:

$$f(\lambda) = \frac{1}{2}a^2\lambda^2K_\nu(\nu),$$

where $K_\nu(x)$ is a modified Hankel function of order 1.

The spectral coefficients, which, according to such correlation function, are determined by calculating the integral:

$$b_k(r) = 2\int_0^\infty J_\nu(r \omega \mu) d\omega = \frac{a^2\lambda^2}{r^2} I_{\nu+k,2+k,2+k+1} K_{\mu+1,1} \frac{r^2}{a^2},$$

Thus the spectral coefficients, that correspond to the Cauchy correlation function (2), are written as follow:

$$b_k(r) = \frac{2^{1+2k}r^{2k}a}{(a+\sqrt{a^2+4r^2})^{2k}} \frac{a^2+4r^2}{(a^2+4r^2)^{2k}} \lambda^2.$$  (5)

These spectral coefficients are calculated by Mathematica software.

The model of 2D random field with Cauchy correlation function and the numerical simulation procedure. The realizations of 2D random field with Cauchy correlation function (3) at the values of parameters $a=1$ are generated. The statistical simulation was performed by the technique of spectral decomposition and finding of spectral coefficients.

From the spectral theory [11] it follows that the model of random fields on a plane with such correlation function is a sum of:

$$\xi_n = \sum_{k=0}^{\infty} \sqrt{\nu_k} b_k(r) \left[ \zeta_k(r) \cos \phi + \eta_k(r) \sin \phi \right]  (6)$$

where:

$$\nu_k = \begin{cases} 1, & k = 0; \\ 2, & k > 0. \end{cases}$$

Define dependence number $N$ on $r$ and $\epsilon$ in the case of Cauchy correlation function (3). It is necessary to calculate the values of $\mu_\nu, K = 1.2$ for the inequality (7), by using the density of distribution (4) and the following formula 6.561(16) [5]:

$$\int_0^\infty x^n K_\nu(ax)dx = 2^{-n}a^{-n-1}\Gamma\left(\frac{1+\mu+n}{2}\right)\Gamma\left(\frac{1+\mu-n}{2}\right).$$
Then the calculated values hold:

\[ \mu_1 = \frac{a^3}{2} \int_{0}^{\infty} \lambda^2 K_i(a \lambda) d \lambda = \frac{3\pi}{4a}, \quad \mu_2 = \frac{a^3}{2} \int_{0}^{\infty} \lambda^4 K_i(a \lambda) d \lambda = \frac{8}{a^2}. \]

Consequently the estimate of the mean square approximation of the random field \( \xi(r, \phi) \) with Cauchy correlation function (3) by the partial sums \( N_{\xi}(r, \phi) \) has the following representation:

\[ N(r, \varepsilon) \geq \frac{1}{\pi} \left( \frac{1}{2} \mu_1 + r^2 \mu_2 \right) = \frac{3\pi}{8a} + \frac{8r^2}{a^2}. \tag{8} \]

The statistical simulation procedure of Gaussian homogeneous isotropic random field \( \xi(r, \phi) \) on the plane was built by means of the model (6) and the estimate (8). This random field is determined by its statistical characteristics: the mathematical expectation and the Cauchy correlation function \( B(r) \) (3) at the value of parameter \( a = 1 \).

Procedure:

1) The positive integer number \( N \) is determined corresponding to the prescribed accuracy \( \varepsilon \) and by using inequality (8), where \( r \) is a radius of the point on the plane in which the realization of the random field \( \xi(r, \phi) \) is generated. The integer number \( N \) equals 59 by using the prescribed accuracy \( \varepsilon = 5 \times 10^{-2} \) and values of parameters \( \nu = 2, \quad a = 1 \).

2) We calculated the spectral coefficients at the value of parameter \( a = 1 \):

\[ b_k(r) = \frac{2^{1-2k} r^{2k}}{(1 + \sqrt{1 + 4r^2})^{2k}} \cdot \frac{(1 + 2r^2 + k \sqrt{1 + 4r^2})}{(1 + 4r^2)^{k/2}}. \]

3) We generate values of the standard normal random variables \( \xi_k, \quad k = 0, 1, 2, \ldots, 59 \) and \( \eta_k, \quad k = 0, 1, 2, \ldots, 59 \).

4) We evaluate the expression (6) by substituting in it values which were found in the previous steps \( q_i = i \times 2\pi \frac{2}{10}, \quad i = 0, 1, 2, \ldots, 9, \quad r_i = 0.1 \times i, \quad i = 1, 2, \ldots, 10 \).

5) The statistical estimate of the correlation function is obtained by the realizations of the random \( \xi(r, \phi) \). This estimate compares with a given correlation function at \( a = 1 \) and provides the statistical analysis the adequacy of realization.

Note that the procedure can be applied to random fields with another type of distribution. Then the random variables \( \{ \xi_k(r), \quad k = 0, 1, 2, \ldots, N \} \) and \( \{ \eta_k(r), \quad k = 0, 1, 2, \ldots, N \} \) should be distributed by corresponding law.

The original Spectr software, based on the results of the statistical data processing and the mentioned procedure for the simulation values of such data realization in the two-dimensional case, was developed in Delphi, where selected Cauchy correlation function (2) was used.

The results, which were obtained by the simulating procedure, are displayed in Figure 5. Figure 5 (a) presents an example of constructed map of chalky strata density according to data boreholes observations (average data over the years to 29 boreholes at 28 m) by Surfer software. Using available data, the accuracy of this construction cannot provide a reliable characteristic of the chalky strata status, because the number of measurement results is not sufficient.

Fig. 5 (b) presents the contours of equal values of chalky strata density based on simulated data including values of the anchor boreholes by means of calculating the spectral coefficients. Additionally, the output data (160 simulated values in intervals between the observation points of this level) can have more reliable approximation that enables more informed decisions about the status of chalky strata and determines places for testing and additional research.

![Fig.5. The distribution of chalky strata density on the industrial area of Rivne nuclear power plant at a depth of 28 m. from the surface, according to the average data of 29 observational boreholes over 1984-2004 years.](image-url)

(a) for the simulated data based on the values in secure boreholes by spectral coefficients (b)
The following Fig. 6 presents the plot of the variogram of the separated random data component of chalky strata density according to Cauchy correlation functions (2) (Fig. 6, (a)) and plot of the variogram of the simulated random data component according to Cauchy correlation functions (2) (Fig. 6, (b)).

\[
\begin{align*}
\text{Fig. 6. The variogram (a) of separated random data component of the chalky strata density,} \\
&\text{corresponding to Cauchy correlation function } B(\rho) = \frac{a^4}{(a^2 + \rho^2)^2}, \ a = 1; \\
&\text{(b) of simulated random data component, corresponding to Cauchy correlation function } B(\rho) = \frac{a^4}{(a^2 + \rho^2)^2}, \ a = 1
\end{align*}
\]

The results present that the chosen model of the data is rather adequate. The developed Spectr2_1 software works with sufficient accuracy.

Conclusions. The theory, techniques and procedure of statistical simulation of 2D random fields can significantly increase the effectiveness of monitoring observations on the territory of potentially dangerous objects. This makes it possible to simulate the values in the area between anchor observation grid and beyond, and to adequately describe real geological processes.

The method of statistical simulation of random fields with Cauchy correlation functions allows complementing the data with a given accuracy. It can also be used to detect abnormal areas.
СТАТИСТИЧНЕ МОДЕЛЮВАННЯ ДВОВИМІРНОГО ВИПАДКОВОГО ПОЛЯ З КОРЕЛЯЦІЙНОЮ ФУНКЦІЄЮ ТИПУ КОШІ В ГЕОФІЗИЧНІЙ ЗАДАЧІ МОНІТОРИНГУ ДОВКІЛЛЯ

У зв'язку з ростом кількості природно-техногенних катастроф актуально є розробка систем моніторингу за станом геологічно-середовища з використанням сучасного математичного апарату та інформаційних технологій. В загальній системі моніторингу дотримується важливою складовою є локальна моніторинга території розташування потенційно небезпечних об'єктів.

На території розміщення Рівненської АЕС проводився комплекс геофізичних досліджень. Серед цих моніторингових спостережень найбільш інтерес представляють радіозалізотопні дослідження густини та вологості ґрунтів по периметру збудованих споруд. При цьому виникає проблема доповнення даних шляхом моделювання, яке проводиться при контролі змін густини крейдяної тошці на території досліджуваного проміжнівника з використанням радіозалізотопних методів по сітці, що включала 29 спостережень. Ця задача в роботі було вирішено методом статистичного моделювання, який надає можливість відображати явище (випадкове поле на площині) у будь-якій точці області спостережень. При цьому моделювання передбачає вивчення значення густини крейдяної тошці на території проміжнівника з використанням побудованої моделі залученням кореляційної функції Коші.

В цій роботі представлено розроблений метод, алгоритм та приклад статистичного моделювання карстово-суфозійних явищ у задачі моніторингу густини крейдяної тошці на території Рівненської АЕС. За спектральним розкладом побудовано статистичну модель розкладу висередненої густини тошці на площині та розроблено алгоритм статистичного моделювання з використанням кореляційної функції Коші. На базі розробленого програмного забезпечення отримано модельні параметри густини крейдяної тошці на сітці спостережень необхідної детальності та регулярності. Проведено статистичний аналіз результатів численного моделювання та їх перевірку на адекватність.

Ключові слова: статистичне моделювання, функція Коші, спектральний розклад, кондиційність карт.